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Technical Note

Uniform transpiration effect on combined heat and mass transfer by natural convection over a cone in saturated porous media: uniform wall temperature/concentration or heat/mass flux

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1. Introduction

Coupled heat and mass transfer (or double-diffusion) driven by buoyancy, due to temperature and concentration variations in a saturated porous medium, has several important applications in geothermal and geophysical engineering, for example, the migration of moisture in fibrous insulation and the underground disposal of nuclear wastes. Comprehensive review on this phenomena has been recently reported by Trevisan and Bejan [1] for various geometries. Bejan and Khair [2] investigated the vertical natural convection boundary layer flow in a saturated porous medium due to the combined heat and mass transfer. Jang and Chang [3] studied the buoyancy-induced inclined boundary layer in a porous medium resulting from combined heat and mass buoyancy effects. Heat and mass transfer about a vertical cylinder in saturated porous media is analyzed by Yücel [4,5]. Lai et al. [6] and Lai and Kulacki [7] performed the cases of slender bodies of revolution and sphere, respectively. Nakayama and Hossain [8] and Singh and Queeny [9] used an integral method to solve the problem of Bejan and Khair [2]. Previous researches [1–9], however, have only concentrated upon the problem of impermeable surface. The suction and blowing effects on steady two-dimensional free convection coupled heat and mass transfer flow through a porous medium is studied by Raptis et al. [10] and Lai and Kulacki [11], respectively.

The objective of the present work, therefore, is to consider the effect of uniform transpiration rate on the heat and mass transfer characteristics in natural convection flow over a cone subjected to uniform wall temperature/concentration or uniform heat/mass flux embedded in porous media under the coupled heat and mass diffusion.

2. Analysis

Consider the problem of the uniform transpiration effect on combined heat and mass free convection flow over a cone embedded in a saturated porous medium. We consider two different conditions at the surface, namely, (I) a uniform wall temperature/concentration (UWT/UWC) and (II) a uniform heat/mass flux (UHF/UMF). Fig. 1 shows the flow model and physical coordinate system. The origin of the coordinate system is placed at the vertex of the cone, where x and y are Cartesian coordinates measuring distance along and normal to the surface of cone, respectively. All the fluid properties are assumed to be constant, except for density variations in the buoyancy term. Introducing the boundary layer and Boussinesq approximations, the governing equations based on the Darcy law can be written as follows:

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial y}=\frac{g\ \cos\ \gamma K}{\nu}\left(\beta_{\mathrm{T}}\frac{\partial T}{\partial y}+\beta_{\mathrm{c}}\frac{\partial c}{\partial y}\right) \tag{2}$$

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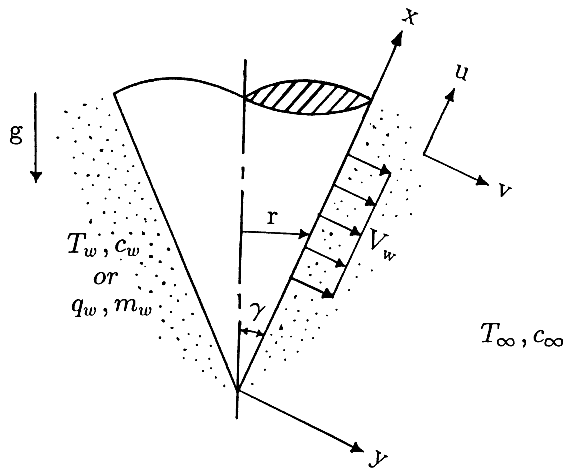

Fig. 1. Flow model and physical coordinate system.

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = D\frac{\partial^2 c}{\partial y^2}. \tag{4}$$

The boundary conditions are defined as follows:

$$y = 0: \quad v = V_\mathrm{w}, \quad T = T_\mathrm{w}, \quad c = c_\mathrm{w}, \tag{5a–c}$$

$$y \to \infty: \quad u = 0, \quad T = T_\infty, \quad c = c_\infty. \tag{6a–c}$$

If the uniform heat flux q_w and uniform mass flux m_w are prescribed, Eqs. (5b,c) are replaced by

$$-k\left(\frac{\partial T}{\partial y}\right)_{y=0} = q_\mathrm{w}, \quad -D\left(\frac{\partial c}{\partial y}\right)_{y=0} = m_\mathrm{w}. \tag{7}$$

We assumed the boundary layer to be sufficiently thin in comparison with the local radius of the cone. The local radius to a point in the boundary layer, therefore, can be replaced by the radius of the cone r, i.e., $r = x \sin\gamma$.

Case 1. Uniform wall temperature/concentration (UWT/UWC)

Invoking the following dimensionless variables

$$\xi = \frac{2V_\mathrm{w}x}{\alpha\, Ra_x^{1/2}}, \quad \eta = \frac{y}{x}\, Ra_x^{1/2}, \quad f(\xi,\eta) = \frac{\psi}{\alpha r\, Ra_x^{1/2}},$$

$$\theta(\xi,\eta) = \frac{T - T_\infty}{T_\mathrm{w} - T_\infty}, \quad C(\xi,\eta) = \frac{c - c_\infty}{c_\mathrm{w} - c_\infty} \tag{8}$$

and substituting Eq. (8) into Eqs. (1)–(6), we obtain

$$f' = \theta + NC, \tag{9}$$

$$\theta'' + \frac{3}{2}f\theta' = \frac{\xi}{2}\left(f'\frac{\partial\theta}{\partial\xi} - \theta'\frac{\partial f}{\partial\xi}\right), \tag{10}$$

$$\frac{1}{Le}C'' + \frac{3}{2}fC' = \frac{\xi}{2}\left(f'\frac{\partial C}{\partial\xi} - C'\frac{\partial f}{\partial\xi}\right). \tag{11}$$

The boundary conditions are defined as follows:

$$\eta = 0: \quad f = -\frac{\xi}{4}, \quad \theta = 1, \quad C = 1, \tag{12a}$$

$$\eta \to \infty: \quad \theta = 0, \quad C = 0. \tag{12b}$$

Here,

$Ra_x = g\ \cos\ \gamma\beta_\mathrm{T}(T_\mathrm{w} - T_\infty)Kx/(\nu\alpha)$

(modified local Rayleigh number).

$N = \beta_\mathrm{c}(c_\mathrm{w} - c_\infty)/[\beta_\mathrm{T}(T_\mathrm{w} - T_\infty)]$ (buoyancy ratio) and

$Pr = \nu/\alpha$ (Lewis number).

Results of practical interest are both heat and mass transfer rates. The local Nusselt number Nu_x and the local Sherwood number Sh_x are, respectively, given by

$$Nu_x/Ra_x^{1/2} = -\theta'(\xi,0), \quad Sh_x/Ra_x^{1/2} = -C'(\xi,0). \tag{13}$$

Case 2. Uniform heat/mass flux (UHF/UMF)

Introducing the following dimensionless variables

$$\xi = \frac{2V_\mathrm{w}x}{\alpha(Ra_x^*)^{1/3}}, \quad \eta = \frac{y}{x}(Ra_x^*)^{1/3}, \quad f(\xi,\eta) = \frac{\psi}{\alpha r(Ra_x^*)^{1/3}},$$

$$\theta(\xi,\eta) = \frac{(T - T_\infty)k(Ra_x^*)^{1/3}}{q_\mathrm{w}x}, \quad C(\xi,\eta) = \frac{(c - c_\infty)D(Ra_x^*)^{1/3}}{m_\mathrm{w}x} \tag{14}$$

and inserting Eq. (14) into Eqs. (1)–(7), we have

$$f' = \theta + N^*C, \tag{15}$$

$$\theta'' + \frac{5}{3}f\theta' - \frac{1}{3}f'\theta = \frac{1}{3}\xi\left(f'\frac{\partial\theta}{\partial\xi} - \theta'\frac{\partial f}{\partial\xi}\right) \tag{16}$$

,

$$\frac{1}{Le}C'' + \frac{5}{3}fC' - \frac{1}{3}f'C = \frac{1}{3}\xi\left(f'\frac{\partial C}{\partial\xi} - C'\frac{\partial f}{\partial\xi}\right). \tag{17}$$

The boundary conditions are defined as follows:

$$\eta = 0: \quad f = -\frac{\xi}{4}, \quad \theta' = -1, \quad C' = -1 \tag{18a}$$

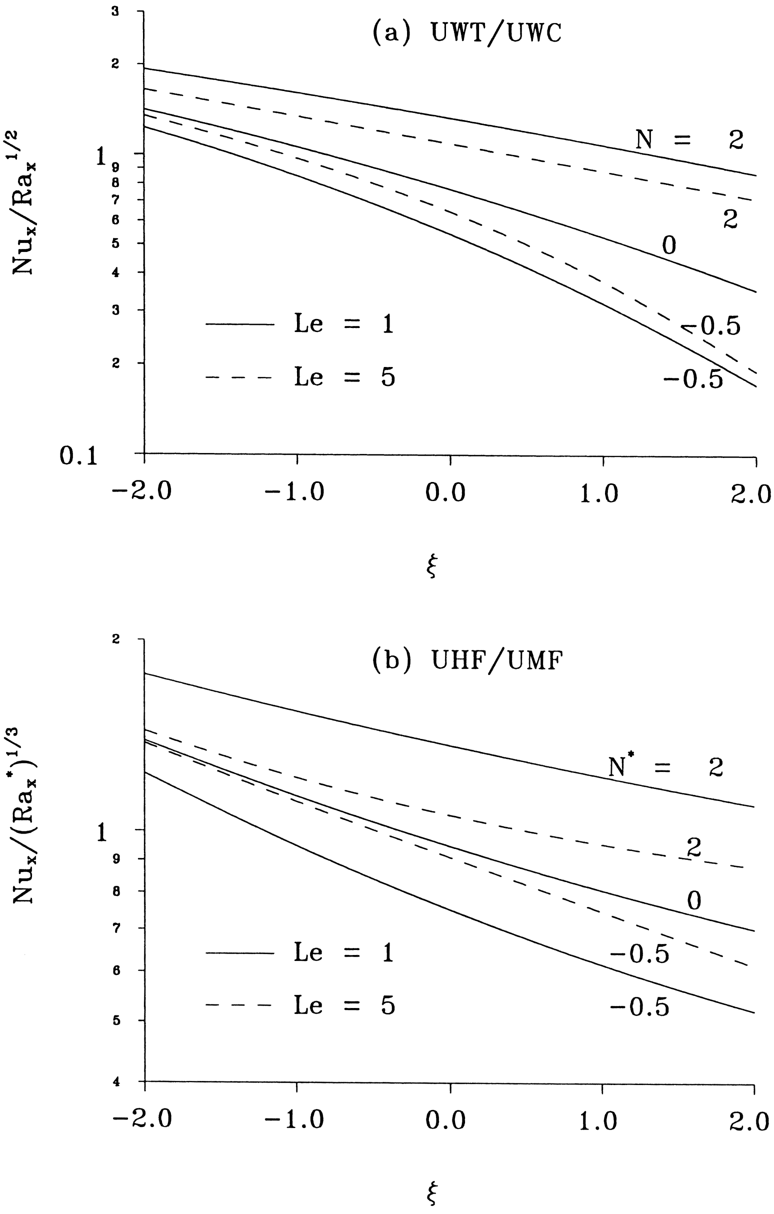

Fig. 2. (a) Local Nusselt number for various values of *Le* and *N* (UWT/UWC). (b) Local Nusselt number for various values of *Le* and N^* (UHF/UMF).

$$\eta \to \infty: \quad \theta = 0, \quad C = 0. \tag{18b}$$

Here,

$Ra_x^* = g \cos \gamma \beta_T q_w K x^2/(\nu \alpha k)$

(modified local Rayleigh number).

$N^* = \beta_c (m_w/D)/[\beta_T (q_w/k)]$ (buoyancy ratio).

The local Nusselt number Nu_x and the local Sherwood number Sh_x can be expressed as

$$Nu_x/(Ra_x^*)^{1/3} = \frac{1}{\theta(\xi,0)}, \quad Sh_x/(Ra_x^*)^{1/3} = \frac{1}{C(\xi,0)}. \tag{19}$$

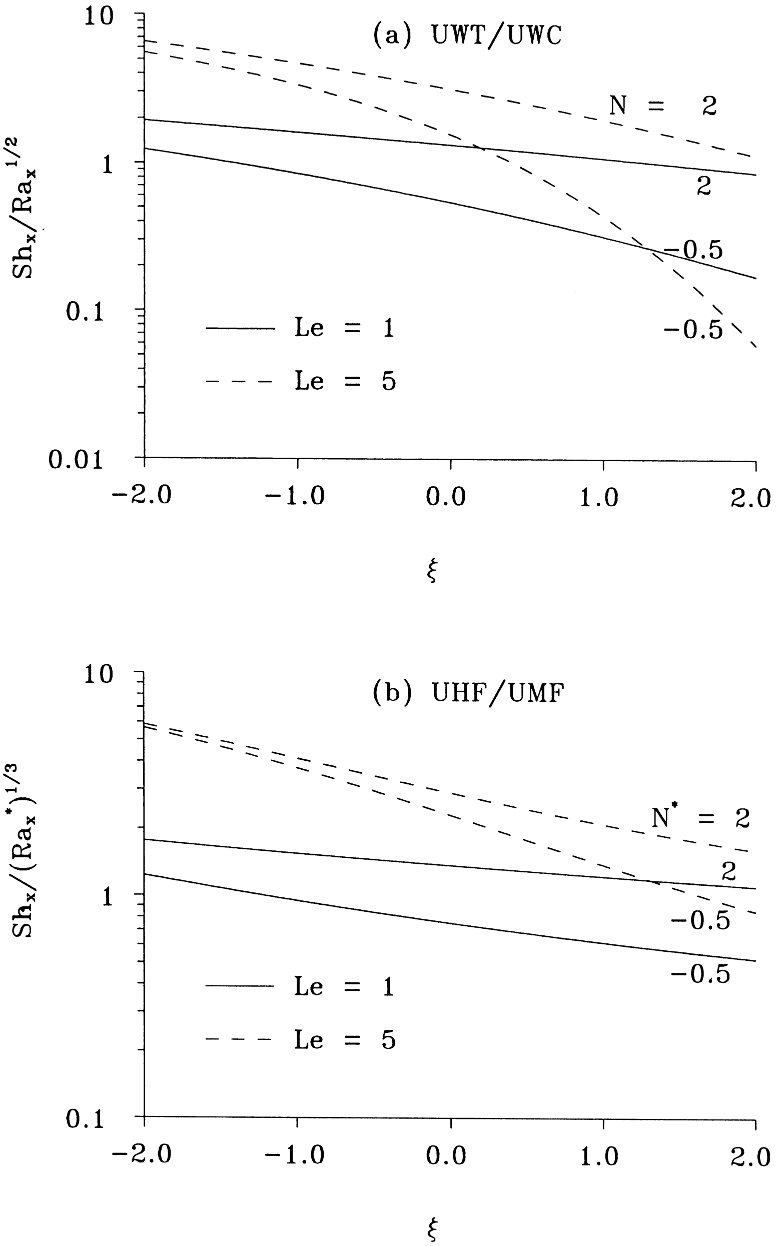

Fig. 3. (a) Local Sherwood number for various values of *Le* and *N* (UWT/UWC). (b) Local Sherwood number for various values of *Le* and N^* (UHF/UMF).

3. Results

Eqs. (9)–(11) and (15)–(17) with the boundary conditions (12) and (18) have been solved numerically using a very efficient implicit finite-difference method known as the modified Keller-box scheme [12]. The details of the solution method are omitted here to conserve space.

Fig. 2 gives the local Nusselt number for various values of *Le* and *N* (or N^*) with the cases of UWT/UWC and UHF/UMF, respectively. It is revealed that the heat transfer rate increases as buoyancy ratio *N* (or N^*) increases or transpiration parameter ξ decreases, for a given value of *Le*. For a given positive *N* (or

N^*), as the Lewis number increases from 1 to 5, the local Nusselt number decreases for all ξ. This is due to the fact that a larger Lewis number *Le* is associated with a thicker thermal boundary layer. The thicker the thermal boundary layer thickness is, the smaller the local Nusselt number is. Whereas, for a given negative N (or N^*), as the Lewis number increases from 1 to 5, the local Nusselt number increases for all ξ.

Fig. 3 shows the local Sherwood number for various values of *Le* and N (or N^*) with the cases of UWT/UWC and UHF/UMF, respectively. It is observed that the mass transfer rate enhances as buoyancy ratio N (or N^*) increases or transpiration parameter ξ decreases, for a fixed value of *Le*. For a given positive N (or N^*), as the Lewis number increases from 1 to 5, the local Sherwood number increases for all ξ. This is because a larger Lewis number *Le* is associated with a thinner concentration boundary layer. The thinner the concentration boundary layer thickness, the greater the local Sherwood number.

4. Concluding remarks

A boundary layer analysis is presented to study the effect of uniform transpiration velocity on natural convection flow in a saturated porous medium resulting from combined heat and mass buoyancy effects adjacent to a cone maintained at a uniform wall temperature/concentration or a uniform heat/mass flux. Numerical solutions are obtained for different values of transpiration parameter, buoyancy ratio, and the Lewis number. It is shown that for the case of suction both the heat and mass transfer rates increase. It is also found that increasing the buoyancy ratio parameter increases the local Nusselt number and Sherwood number. As the Lewis number increases, the local Nusselt (Sherwood) number decreases (increases).

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